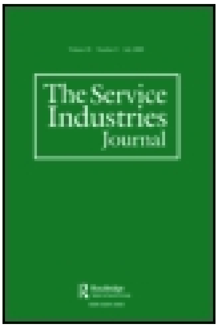

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Pricing policies for services by facilities with Weibull lifetimes

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To make good profits, pricing is a competitive weapon of service firms. This paper is concerned with pricing strategies for services with substantial facility maintenance costs. To address the problem, a mathematical framework that incorporates service demand and facility deterioration is proposed. The facility and customers constitute a service system driven by Poisson arrivals and exponential service times. The most common log-linear customer demand and Weibull-distributed facility lifetime are also adopted. By examining the linkage between customer demand and facility deterioration in profit model, pricing policies of the service are investigated. Then analytical conditions of customer demand and facility lifetime are derived to achieve a unique optimal pricing policy. Finally, numerical examples are presented to illustrate the effects of parameter variations on the optimal pricing policy.

Keywords: service pricing; facility maintenance; Poisson process; log-linear demand; Weibull lifetime

Introduction

The service sector occupies a large portion of economy in most developed countries and hence has received increasing attention. In the meanwhile, service firms face many challenges for survival and success. Pricing as a competitive weapon to attract customers is an essential issue for most service firms. In this paper, our focus is restricted to services delivered to customers by the operation of facilities. These kinds of services are common and important in many service industries. In practice, especially for capital-intensive industries such as healthcare, transportation, and telecommunications industries, the service facilities need to be repaired upon failures and thus substantial costs are ascribed to facility maintenance.

For example, the radiology department of a medical centre usually has advanced imaging facilities such as computed tomography and magnetic resonance imaging equipment. Most technical revenues of the department are earned from advanced imaging examinations provided by these facilities. However, in addition to heavy capital investments upon procurement, this kind of medical equipment requires a large portion of total costs

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to be spent on equipment maintenance to ensure smooth operation and service quality. More precisely, during the medical examination process of a patient, the medical equipment deteriorates and might fail because of its usage. Although this is an illustration in healthcare institutions, from the perspective of facility maintenance, there are many similar scenarios in other service industries.

Due to the feature in cost structures, service firms of the type characterized above should examine facility maintenance costs in detail to determine competitive pricing policies for good profits. However, in contrast to the importance of these service industries, service pricing with considerable maintenance costs has received relatively little attention. This necessity inspires our study to fill the vacuum in the existing literature.

Therefore, this paper strives to achieve the following objectives: (1) to identify the relationship between service demand and facility deterioration; (2) to propose a model that incorporates revenues of customer service and costs of facility maintenance; and (3) to assist the service firms concerned to develop appropriate pricing strategies subject to facility maintenance.

The rest of this paper is organized as follows. Two streams of relevant literature are briefly reviewed in the second section. The mathematical model of the service firm's profits with facility maintenance costs is formulated in the third section. According to the profit model, a unique optimal pricing policy for the service is achieved in the fourth section. The structural properties of the optimal pricing policy are also investigated and discussed in the fourth section. The effects of parameter variations on the optimal pricing policy are illustrated by numerical examples in the fifth section. Finally, concluding remarks and suggestions for further developments are summarized in the last section.

Literature review

There are two streams of literature relevant to this paper. One is service pricing, the other is maintenance planning. Relevant literature on service pricing is briefly introduced as follows. In most of the previous works, pricing is regarded as a mechanism to obtain social optimization, admission control, and reduction of waiting costs for service systems. Extensive discussions about pricing policies of services can be found in Hassin and Haviv (2003).

Two more related studies to this paper are So and Song (1998) and Ziya, Ayhan, and Foley (2006). The models in these two referred papers have similarities with our framework in the perspective of customer demand. So and Song investigate the joint policies of price, delivery time guarantee, and capacity level of an average per-unit-time profit. In their framework, delivery time is regarded as a signal of service quality so that delivery time guarantee can also be used as a strategy to attract customers. Therefore, they treat demand rate as a function of both price and delivery time guarantee and then adopt a bi-variable log-linear demand to reflect customer sensitivity in both variables. Ziya et al. (2006) explore pricing policies for finite capacity systems under an increasing price elasticity, which is also assumed in our paper. Instead of using log-linear demand, Ziya et al. employ a random service valuation by customers to depict the relation between customer arrivals and service price. The price elasticity is thus given by a willingness-to-pay distribution. However, in all the aforementioned studies, the service facility is assumed to be reliable and needs no maintenance actions. Therefore, additional maintenance theory of service facilities is essential to resolve our proposed problem.

Relevant literature on maintenance planning is briefly reviewed as follows. Most of the earlier studies investigate schemes for appropriate maintenance actions to ensure smooth operation of facilities. Comprehensive reviews of various maintenance policies are referred to Barlow and Proschan (1965), Scarf (1997), and Wang (2002). There are two main types of maintenance for repairable facilities: corrective maintenance (CM) and preventive maintenance (PM). CM actions rectify a failed facility to restore its operation; PM actions are arranged and performed to reduce failures of an operational facility. Among all types of CM, minimal repair is the most widely adopted maintenance action. Immediately after minimal repair, the failure rate of the facility is the same as just before failure. In other words, the rectified facility is identical to what it was just before failure (as bad as the old). The type of PM employed in our study is periodic perfect PM, which is performed periodically and brings the facility back to an as good as new state. Compared with most other types of PM, periodic perfect PM is relatively simple and easy to be implemented in practice (Barlow & Hunter, 1960; Nakagawa & Kowada, 1983).

Since this paper deals with facilities in service systems, those studies devoted to intermittently operated facilities are more closely related to our work. Koyanagi and Kawai (2003) explore optimal observation time for PM actions in an *M*/*G*/*l* queueing system. Hsu (1992, 1999) establishes schedules of maintenance actions according to the number of items produced in a queue-like production system. Dohi, Kaio, and Osaki (2001) employ renewal theory to derive optimal PM policies over infinite time horizons under an intermittently used environment. In Koyanagi and Kawai (2003), the ageing process of the facility is assumed to continue during idle time. This measure of facility failures might be suitable for some facilities like vending machines. However, it would not be appropriate for the scenarios in this paper. In Hsu (1992, 1999) and Dohi et al. (2001), the facility age is measured in cumulative operating time, not in calendar time. This approach only applies to a facility with a time meter that records the cumulative operating time. It is not practical either to service systems whose opening hours must be regular according to calendar time. In the service system of this study, there is no facility deterioration during idle time and all system operations are conducted according to calendar time. A measure of facility failures that fits our need is left to be derived. This brief review of these two streams of previous studies identifies the gap to be bridged between them. Therefore, this paper will make contributions to service pricing with the emphasis on maintenance costs of the service facilities subject to failures.

Model formulation

In this section, basic concepts and assumptions of the model are introduced. Consider a service system in which the service is provided by an operating facility. Each arriving customer from the Poisson stream is served an exponential time. Furthermore, a significant portion of total costs is spent on facility maintenance to ensure smooth operation of the service system.

According to the framework illustrated in Figure 1, the primary activities relevant to the profits of the service firm are described as follows. First, the service demand is realized by a pricing policy. Next, according to a Poisson process governed by the demand rate, customers arrive to acquire service provided by the facility. Finally, customers are served and thus the facility deteriorates because of usage.

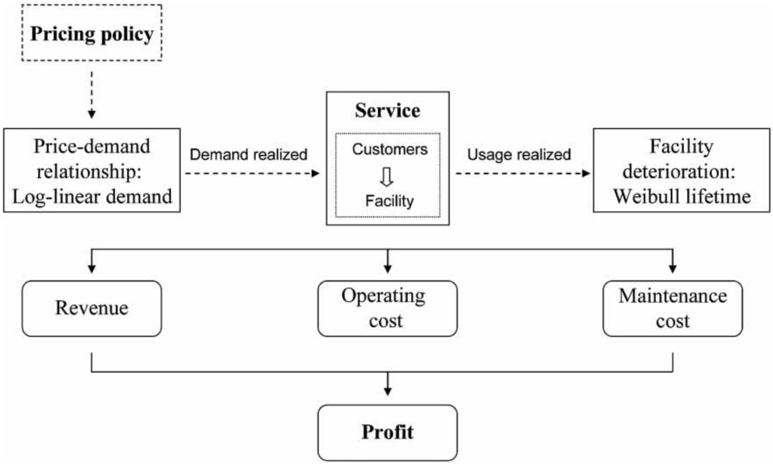

Figure 1. Model framework.

As shown in Figure 1, the profits corresponding to the aforementioned activities primarily consist of revenues of sold services, unit operating costs, and maintenance costs of the facility. In particular, maintenance costs of the facility are incurred by failures and deterioration. Facility failures are rectified immediately by minimal repairs and facility condition is brought back to as good as new by PM in the setup for each working period. In this system, setup cost is primarily the expenditure incurred by PM to initiate a working period.

Since service demand brings facility usage and then the induced maintenance actions bring a significant portion of total costs, service demand and facility lifetime are thus interrelated. Therefore, to maximize the total cash flow depicted in Figure 1, the service firm should carefully examine the linkage between service demand and facility deterioration in the determination of pricing policies. Before proceeding further, we need to introduce the notations and assumptions employed in this paper.

Notations and assumptions

The decision variable is the service price which is denoted by p. Other notations used to construct the profit model for the service firm are summarized as follows.

λ	demand rate of the service
μ	mean processing rate of the service
$F(t)$	lifetime distribution of the facility
$h(t)$	failure (hazard) rate function of the facility
$H(t)$	cumulative failure (hazard) rate function of the facility
T	duration between two successive facility setups; length of working period
$K(\lambda;T)$	number of customers in the time interval $[0, T]$
$N(\lambda;T)$	number of facility failures in the time interval $[0, T]$
$M(\lambda;T)$	expected value of $N(\lambda;T)$

c	unit operating cost of the service (e.g. material and direct labor costs)
C_{m}	CM (minimal repair) cost of the facility
C_p	facility setup cost for each working period of the service
π	expected profit of the service firm

To capture analytical insights from the mathematical model, Assumptions A1–A5 are employed to avoid tedious technicalities. Specifications on service demand and facility deterioration are described afterwards.

A1. The system has no facility deterioration during idle time.

A2. Time to carry a CM/PM action is negligible relative to a working period.

A3. A CM action is performed immediately after each facility failure.

A4. Each customer entering the system will complete the service, possibly beyond the official opening hours of the service firm. (The actual service period might exceed the official opening hours.)

A5. The system satisfies a light traffic condition, that is, $\lambda/\mu < 1$.

A6. Service time is cumulative after facility maintenance: the customer service which is interrupted by a facility failure will be resumed immediately after the completion of facility maintenance.

The service demand

To characterize customer sensitivity in service price, the demand rate for the service is assumed to be driven by the log-linear (Cobb–Douglas) demand

$$\lambda = \lambda_0 p^{-\varepsilon}, \tag{1}$$

where $\lambda_0 > 0$ is the potential customer arrival rate. While simple, the log-linear model admits many realistic scenarios in several service industries such as healthcare, telecommunications, and information technology (Brynjolfsson, Hu, & Smith, 2003; Gyldmark & Morrison, 2001; Lanning, Mitra, Wang, & Wright, 2000).

By definition, the parameter ε is exactly the price elasticity of the service. In practice, the demands of services/goods are generally decreasing and convex with respect to their prices. Clearly, the price elasticity ε must be positive so that the demand rate λ is decreasing and convex in the price p. Furthermore, $\varepsilon > 1$ is assumed throughout this paper so that the demand rate is also price elastic. This assumption means a price increase will result in a decrease in revenues, which implies that the market power of the service firm is limited (Lanning et al., 2000; So & Song, 1998).

The facility lifetime

To incorporate the maintenance costs incurred by facility deterioration into the pricing decision, the lifetime of the facility in a continuously used environment is assumed to be Weibull-distributed and thus its probability density function can be written as $f(t) = \alpha\beta(\alpha t)^{\beta-1}e^{-(\alpha t)^{\beta}}$, $t > 0$, where $\alpha > 0$ is the scale parameter and $\beta > 0$ is the shape parameter. Then it follows from definitions that the failure rate function is

$$h(t) = \alpha\beta(\alpha t)^{\beta-1} \tag{2}$$

and the cumulative failure rate function is

$$H(t) = (\alpha t)^{\beta}. \tag{3}$$

Note that greater value of β indicates faster deterioration of the facility. More precisely, the failure rate h is increasing for $\beta > 1$, decreasing for $\beta < 1$, and constant for $\beta = 1$. Since an increasing failure rate corresponds to a deteriorating facility which is considered in most cases, $\beta > 1$ is assumed throughout this paper.

It is also worth mentioning that the Weibull distribution is the most widely adopted lifetime distribution model in the literature. Because of various shape and scale parameters, the Weibull distribution can describe or approximate diverse types of lifetimes (Lawless, 2003).

The profit model

In this section, the scenario and assumptions are combined together to formulate the mathematical model. For each working period of the service, the profits of the service firm consist of four major components: service revenue, operating cost, CM cost, and setup cost. In terms of the notations defined in Notations and assumptions, the four profit components can be mathematically expressed as follows: the total service revenue is $pK(\lambda;T)$; the total operating cost is $cK(\lambda;T)$; the total CM cost is $C_mN(\lambda;T)$; and the facility setup cost is C_p. Combining the four major components, the profit function of the service firm is formulated as

$$\pi^0(p) = (p - c)K(\lambda; T) - C_mN(\lambda; T) - C_p.$$

Since the number of customers $K(\lambda;T)$ follows the distribution Poisson $(T\lambda)$ from the Poisson arrival assumption, taking the expectation of $\pi^0(p)$ yields the expected profit function

$$\pi(p) = E[\pi^0(p)] = (p - c)T\lambda - C_mM(\lambda; T) - C_p. \tag{4}$$

Now the primary objective of the service firm is to find optimal pricing policies p^* to maximize the expected profit (4).

Model analysis

In this section, a unique optimal pricing policy of the service is derived. Obviously the expected number of facility failures $M(\lambda;T)$ plays an important role in the profit model (4). Hence several analytical properties of $M(\lambda;T)$ are presented and discussed before the exploration of the whole profit model.

Analytical properties of the expected number of facility failures

The expected number of facility failures is determined by the lifetime distribution and maintenance policy of the facility. In a continuously operated environment without PM actions, because facility failures are rectified by minimal repairs, in terms of the failure rate h, the failure process of the facility is a nonhomogeneous Poisson process with intensity h (Nakagawa & Kowada, 1983). Hence, the expected number of facility failures during the time interval [0, T] is accordingly given by $\int_0^T h(s)\mathrm{d}s = H(T)$.

However, in this intermittently used environment illustrated in Figure 2, the expected number of facility failures during the time interval [0, T] should be derived in another modified approach. Let T_S denote the total cumulative operating time serving customers during the finite duration [0, T]. Assumption A1 indicates that T_S accounts for all failure occurrences. Therefore, by Nakagawa and Kowada (1983), the conditional expected number of facility failures is $H(t_S)$ for each given $T_S = t_S$. That is, $E[N(\lambda;T)|T_S = t_S] = H(t_S)$. Then the double expectation theorem immediately yields that the expected number of facility failures $M(\lambda;T) = E[N(\lambda;T)] = E[H(T_S)]$. It now turns our focus to the distribution of T_S.

Assumptions A2–A3 imply that the facility is available almost in the whole duration [0, T]. Additionally, Assumptions A4–A6 imply that each customer entering the system will be served an exponential time. Combining these two implications, the total cumulative operating time T_S is approximately $Y_1 + Y_2 + \cdots + Y_{K(\lambda;T)}$, where $Y_i \sim \exp(\mu)$ is the service time for the ith customer. Therefore, the conditional distribution of T_S given $K(\lambda;T) = k$, $k = 1,2, \ldots$, is the Erlang-k distribution with rate μ, that is,

$$f_{T_s}(t)\big|_{[K(\lambda;T)=k]} = \frac{\mu^k}{\Gamma(k)} t^{k-1}\mathrm{e}^{-\mu t} \quad \text{for } t > 0, \tag{5}$$

where $\Gamma(\cdot)$ is the gamma function defined by $\Gamma(z) = \int_0^\infty t^{z-1}\mathrm{e}^{-t}\mathrm{d}t$. From Equations (3) and (5), the conditional expectation of the number of facility failures $N(\lambda;T)$ given $K(\lambda;T) = k$ becomes

$$\begin{aligned} E[H(T_S)|K(\lambda;T) = k] &= E[(\alpha T_S)^\beta|K(\lambda;T) = k] \\ &= \alpha^\beta \int_0^\infty \frac{\mu^k}{\Gamma(k)} t^{\beta+k-1}\mathrm{e}^{-\mu t}\mathrm{d}t = \left(\frac{\alpha}{\mu}\right)^\beta \frac{\Gamma(\beta+k)}{\Gamma(k)}, \end{aligned}$$

where $k = 1,2, \ldots$. Applying the double expectation theorem, the expected number of facility failures can be written as $M(\lambda;T) = E[N(\lambda;T)] = E[E[N(\lambda;T)|K(\lambda;T)]]$ and thus

$$M(\lambda;T) = \left(\frac{\alpha}{\mu}\right)^\beta \sum_{k=1}^\infty \frac{\Gamma(\beta+k)}{\Gamma(k)} \mathrm{e}^{-T\lambda} \frac{(T\lambda)^k}{k!}. \tag{6}$$

Figure 2. Facility usage in the service system.

Here follows the main result of this section which is essential for the investigation of the profit model.

Proposition 1 The expected number of facility failures $M(\lambda;T)$ is strictly increasing and strictly convex in demand rate λ.

Proof

To prove these analytical properties of $M(\lambda;T)$, it suffices to show that $(d/d\lambda)M(\lambda;T) > 0$ and $(d^2/d\lambda^2)M(\lambda;T) > 0$ for all $\lambda > 0$. The two derivatives of $M(\lambda;T)$ can be obtained from Equation (6) by term-by-term differentiation. For the first-order derivative of $M(\lambda;T)$, the calculations are carried out directly below.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\lambda}M(\lambda;T) &= \left(\frac{\alpha}{\mu}\right)^{\beta}\sum_{k=1}^{\infty}\frac{\Gamma(\beta+k)}{\Gamma(k)}T\mathrm{e}^{-T\lambda}\left[\frac{(T\lambda)^{k-1}}{(k-1)!}-\frac{(T\lambda)^{k}}{k!}\right] \\
&= \left(\frac{\alpha}{\mu}\right)^{\beta}T\mathrm{e}^{-T\lambda}\left\{\Gamma(\beta+1)+\sum_{k=1}^{\infty}\left[\frac{\Gamma(\beta+k+1)}{\Gamma(k+1)}-\frac{\Gamma(\beta+k)}{\Gamma(k)}\right]\frac{(T\lambda)^{k}}{k!}\right\} \qquad (7) \\
&= T\beta\left(\frac{\alpha}{\mu}\right)^{\beta}\sum_{k=0}^{\infty}\frac{\Gamma(\beta+k)}{\Gamma(k+1)}\mathrm{e}^{-T\lambda}\frac{(T\lambda)^{k}}{k!}
\end{aligned}$$

in which the simplified expression (7) of $(d/d\lambda)M(\lambda;T)$ follows from the algebraic manipulations

$$\frac{\Gamma(\beta+k+1)}{\Gamma(k+1)}-\frac{\Gamma(\beta+k)}{\Gamma(k)}=\frac{(\beta+k)\Gamma(\beta+k)-k\Gamma(\beta+k)}{\Gamma(k+1)}=\frac{\beta\Gamma(\beta+k)}{\Gamma(k+1)}.$$

For the second-order derivative of $M(\lambda;T)$, similar calculations are also carried out directly below:

$$\begin{aligned}
\frac{\mathrm{d}^2}{\mathrm{d}\lambda^2}M(\lambda;T) &= T^2\beta\left(\frac{\alpha}{\mu}\right)^{\beta}\mathrm{e}^{-T\lambda}\left\{-\Gamma(\beta)+\sum_{k=1}^{\infty}\frac{\Gamma(\beta+k)}{\Gamma(k+1)}\left[\frac{(T\lambda)^{k-1}}{(k-1)!}-\frac{(T\lambda)^{k}}{k!}\right]\right\} \\
&= T^2\beta\left(\frac{\alpha}{\mu}\right)^{\beta}\mathrm{e}^{-T\lambda}\left\{-\Gamma(\beta)+\Gamma(\beta+1)+\sum_{k=1}^{\infty}\left[\frac{\Gamma(\beta+k+1)}{\Gamma(k+2)}-\frac{\Gamma(\beta+k)}{\Gamma(k+1)}\right]\frac{(T\lambda)^{k}}{k!}\right\} \qquad (8) \\
&= T^2\beta(\beta-1)\left(\frac{\alpha}{\mu}\right)^{\beta}\sum_{k=0}^{\infty}\frac{\Gamma(\beta+k)}{\Gamma(k+2)}\mathrm{e}^{-T\lambda}\frac{(T\lambda)^{k}}{k!}
\end{aligned}$$

which is also simplified by the analogous algebraic manipulations

$$\frac{\Gamma(\beta+k+1)}{\Gamma(k+2)}-\frac{\Gamma(\beta+k)}{\Gamma(k+1)}=\frac{(\beta+k)\Gamma(\beta+k)-(k+1)\Gamma(\beta+k)}{\Gamma(k+2)}=\frac{(\beta-1)\Gamma(\beta+k)}{\Gamma(k+2)}.$$

Applying the increasing failure rate assumption, that is, $\beta > 1$, to the two expressions (7) and (8), both derivatives $(d/d\lambda)M(\lambda;T)$ and $(d^2/d\lambda^2)M(\lambda;T)$ are positive and thus the proof is completed.

Note that if β is integer-valued, the factor $\Gamma(\beta + k)/\Gamma(k)$ in Equation (6) reduces to $k(k+1)\cdots(k+\beta-1)$ which is simply a polynomial in k of order β. Also recall the fact that all moments about the origin of a Poisson random variable with parameter $T\lambda$ are polynomials in $T\lambda$ (Johnson, Kemp, & Kotz, 2005). Therefore, $M(\lambda;T)$ can be explicitly expressed as a polynomial in $T\lambda$ of order β provided that β is integer-valued.

Additionally, Proposition 1 reveals the implications that when the service facility is deteriorating over operating time: (a) higher customer frequency results in faster facility deterioration and thus more expected failure occurrences; and (b) as customer frequency increases, the growth rate of expected failure occurrences also increases.

Optimal pricing policy for the service firm

The service firm has to determine a service price p to achieve the maximal profit. However, observations on both structures of the expected profit (4) and the expected number of facility failures (6) suggest that it is more convenient to express the service price p in terms of the demand rate λ. Assume that an optimal demand rate λ^* exists for a moment. Then the optimal price p^* can be obtained by the aforementioned inverse demand relationship $p^* = p(\lambda^*)$. That is, the service firm needs only to operate the system at its optimal customer arrival rate and to post the optimal service price by the price–demand relationship.

By substituting the inverse demand function $p = p(\lambda)$ given by Equation (1) into Equation (4), the expected profit function is equivalently reformulated as

$$\pi(\lambda) = \pi(p(\lambda)) = T[\lambda_0^{1/\varepsilon}\lambda^{1-1/\varepsilon} - c\lambda] - C_m M(\lambda; T) - C_p. \tag{9}$$

Therefore, the original maximization problem is transformed into

$$\begin{aligned} &\max \pi(\lambda) = T[\lambda_0^{1/\varepsilon}\lambda^{1-1/\varepsilon} - c\lambda] - C_m M(\lambda; T) - C_p \\ &\text{subject to } 0 < \lambda < \mu. \end{aligned} \tag{10}$$

The necessary conditions of optimal policies are readily given by the Karush–Kuhn–Tucker conditions. However, these conditions are not practical to locate the optimal policies. Therefore, we will present an alternative to characterize optimal demand rates.

At first, it proceeds in the manner of the traditional approach: the first two derivatives of $\pi(\lambda)$ are calculated as follows to investigate the necessary first-order and second-order conditions for optimality.

$$\frac{d\pi}{d\lambda} = T\left[\left(1-\frac{1}{\varepsilon}\right)\lambda_0^{1/\varepsilon}\lambda^{-1/\varepsilon} - c\right] - C_m\frac{d}{d\lambda}M(\lambda;T) = T\left[\left(1-\frac{1}{\varepsilon}\right)\lambda_0^{1/\varepsilon}\lambda^{-1/\varepsilon} - c\right]$$
$$- C_m T\beta\left(\frac{\alpha}{\mu}\right)^{\beta}\sum_{k=0}^{\infty}\frac{\Gamma(\beta+k)}{\Gamma(k+1)}e^{-T\lambda}\,\frac{(T\lambda)^k}{k!},$$

$$\frac{d^2\pi}{d\lambda^2} = -T\left(\frac{1}{\varepsilon}-\frac{1}{\varepsilon^2}\right)\lambda_0^{1/\varepsilon}\lambda^{-1/\varepsilon-1} - C_m\frac{d^2}{d\lambda^2}M(\lambda;T)$$
$$= -T\left(\frac{1}{\varepsilon}-\frac{1}{\varepsilon^2}\right)\lambda_0^{1/\varepsilon}\lambda^{-1/\varepsilon-1}$$

$$- C_m T^2\beta(\beta-1)\left(\frac{\alpha}{\mu}\right)^{\beta}\sum_{k=0}^{\infty}\frac{\Gamma(\beta+k)}{\Gamma(k+2)}e^{-T\lambda}\frac{(T\lambda)^k}{k!}.$$

Clearly, $\pi(\lambda)$ is globally concave. It is simply because the two assumptions that the service demand is price elastic (i.e. $\varepsilon > 1$) and the facility failure rate h is strictly increasing (i.e. $\beta > 1$) immediately imply that the second-order condition $d^2\pi/d\lambda^2 < 0$ always holds for all λ. Hence the global optimality is assured for a demand rate that satisfies the first-order condition $d\pi/d\lambda = 0$ and the light traffic condition $0 < \lambda < \mu$. To investigate the first-order condition, let an auxiliary function ϕ be defined as

$$\phi(\lambda) = \left[C_m\beta\left(\frac{\alpha}{\mu}\right)^{\beta}\sum_{k=0}^{\infty}\frac{\Gamma(\beta+k)}{\Gamma(k+1)}e^{-T\lambda}\frac{(T\lambda)^k}{k!} + c\right]\lambda_0^{-1/\varepsilon}\lambda^{1/\varepsilon}.$$

Then

$$\phi(\lambda) = \frac{\{(C_m/T)(d/d\lambda)M(\lambda;T) + c\}}{p(\lambda)} \tag{11}$$

and the original first-order condition $d\pi/d\lambda = 0$ is equivalent to

$$\phi(\lambda) = 1 - \frac{1}{\varepsilon}. \tag{12}$$

Note that the first-order condition (12) reveals the trade-off between service revenue and maintenance cost in the determination of optimal pricing policies: as more customers are served, more service revenue is received but higher facility deterioration and thus more CM cost are incurred. On the other hand, as fewer customers are served, less service revenue is received but lower facility deterioration and thus less CM cost are incurred. For convenience, these results obtained so far are summarized in the following lemma.

Lemma 1 If a demand rate λ^* maximizes the problem (10), then $\phi(\lambda^*) = 1 - 1/\varepsilon$.

For further investigation of optimal pricing policies, two analytical properties of the function ϕ are stated immediately below.

Lemma 2 (a) ϕ is positive, strictly increasing, and differentiable for $\lambda \in (0,\mu)$. (b) ϕ admits all values between 0 and $\phi(\mu)$.

Proof

(a) Clearly, ϕ is positive on $(0,\mu)$ by its definition. To complete the proof, it suffices to show that $\mathrm{d}\phi/\mathrm{d}\lambda > 0$ on $(0,\mu)$. Direct differentiation of ϕ yields that

$$\frac{\mathrm{d}\phi}{\mathrm{d}\lambda} = \frac{C_\mathrm{m}}{Tp(\lambda)}\frac{\mathrm{d}^2}{\mathrm{d}\lambda^2}M(\lambda;T) - \frac{1}{p(\lambda)^2}\frac{\mathrm{d}p}{\mathrm{d}\lambda}\left[\frac{C_\mathrm{m}}{T}\frac{d}{\mathrm{d}\lambda}M(\lambda;T)+c\right].$$

Applying Proposition 1 with the fact $\mathrm{d}p/\mathrm{d}\lambda = -(1/\varepsilon)\lambda_0^{1/\varepsilon}\lambda^{-1/\varepsilon-1} < 0$, we have $\mathrm{d}\phi/\mathrm{d}\lambda > 0$.
(b) Since ϕ is strictly increasing and continuous for $\lambda \in (0,\mu]$, by the intermediate value theorem, it suffices to show that $\phi(0^+) = 0$. By Equation (7), $\lim_{\lambda\to 0^+}(\mathrm{d}/\mathrm{d}\lambda)M(\lambda;T) = T\beta(\alpha/\mu)^\beta\Gamma(\beta)$ which exists as a finite value. Substituting this result along with $\lim_{\lambda\to 0^+} p(\lambda)^{-1} = 0$ into the definition of ϕ given by Equation (11), it follows that $\phi(0^+)=0$.

Now, the exploration returns to the existence of the optimal pricing policy. Assume that $\phi(\mu)>1-1/\varepsilon$. Then, by Lemma 2, ϕ must admit the value $1-1/\varepsilon$ at some unique point $\lambda^*\in(0,\mu)$. Hence the first-order condition (12) and the light traffic condition $0<\lambda<\mu$ are both satisfied at λ^*. More precisely, $\pi(\lambda)$ is strictly increasing on $(0,\lambda^*]$, strictly decreasing on $[\lambda^*,\mu)$, and strictly concave on $(0,\mu)$. The preceding arguments lead to the following proposition that characterizes the optimal solution to the maximization problem (10).

Proposition 2 Suppose that $\phi(\mu)>1-1/\varepsilon$. Then the expected profit $\pi(\lambda)$ admits a unique maximum at the demand rate

$$\lambda^* = \phi^{-1}\left(1-\frac{1}{\varepsilon}\right). \qquad (13)$$

Although the auxiliary function ϕ might be complicated so that the optimal demand rate λ^* might admit no closed-form expression, the first-order condition (12) that determines λ^* can be solved numerically by traditional root-finding algorithms such as Newton's method and the secant method.

Comparative statics analysis

The optimal service demand λ^* given by Proposition 2 can be regarded as a function of the parameters in the profit model, while the parameters reflect the business environment. To find proper reaction to environment changes, the shifting behaviors of λ^* should be analyzed. Several comparative statics properties of λ^* are summarized in Proposition 3.

Proposition 3 Suppose that $\phi(\mu) > 1-1/\varepsilon$. Then the optimal service demand λ^* determined by the first-order condition (12) is affected by the model parameters in the following manners:

(a) λ^* is strictly increasing in price elasticity ε for $\log(\lambda_0/\lambda^*) < \varepsilon/(\varepsilon-1)$; λ^* is strictly decreasing in price elasticity ε for $\log(\lambda_0/\lambda^*) > \varepsilon/(\varepsilon-1)$;
(b) λ^* is strictly increasing in potential demand λ_0;
(c) λ^* is strictly increasing in service processing rate μ;
(d) λ^* is strictly decreasing in the length of working period T
(e) λ^* is strictly decreasing in CM cost C_m and unit operating cost c;

(f) λ^* is strictly decreasing in the scale parameter α and shape parameter β of facility lifetime distribution.

Proof

Properties (b), (c), (e), and (f) can be easily verified through the first-order condition (12). Therefore, our task remains to prove properties (a) and (d). Partial differentiation with respect to ε on both sides of Equation (12) yields that

$$\frac{1}{\varepsilon^2} = \frac{\partial \phi}{\partial \varepsilon} + \frac{\mathrm{d}\phi}{\mathrm{d}\lambda}(\lambda^*)\frac{\partial \lambda^*}{\partial \varepsilon} = -\left(\frac{1}{\varepsilon^2}\right)\phi(\lambda^*)\log\left(\frac{\lambda^*}{\lambda_0}\right) + \frac{\mathrm{d}\phi}{\mathrm{d}\lambda}(\lambda^*)\frac{\partial \lambda^*}{\partial \varepsilon}$$
$$= -\left(\frac{1}{\varepsilon^2}\right)\left(1-\frac{1}{\varepsilon}\right)\log\left(\frac{\lambda^*}{\lambda_0}\right) + \frac{\mathrm{d}\phi}{\mathrm{d}\lambda}(\lambda^*)\frac{\partial \lambda^*}{\partial \varepsilon}$$

which is equivalent to

$$\varepsilon^2\frac{\mathrm{d}\phi}{\mathrm{d}\lambda}(\lambda^*)\frac{\partial \lambda^*}{\partial \varepsilon} = 1 + \left(1-\frac{1}{\varepsilon}\right)\log\left(\frac{\lambda^*}{\lambda_0}\right).$$

Since $(\mathrm{d}\phi/\mathrm{d}\lambda)(\lambda^*) > 0$ by Lemma 2, $\partial \lambda^*/\partial \varepsilon > 0$ if and only if $(1 - 1/\varepsilon)\log(\lambda_0/\lambda^*) < 1$ or equivalently $\log(\lambda_0/\lambda^*) < \varepsilon/(\varepsilon - 1)$. This proves property (a). Finally, to prove property (d), the first-order condition (12) is rewritten as

$$C_\mathrm{m}\beta\left(\frac{\alpha}{\mu}\right)^\beta\sum_{k=0}^{\infty}\frac{\Gamma(\beta+k)}{\Gamma(k+1)}\mathrm{e}^{-T\lambda^*}\frac{(T\lambda^*)^k}{k!} + c = \left(1-\frac{1}{\varepsilon}\right)\lambda_0^{1/\varepsilon}(\lambda^*)^{-1/\varepsilon}.$$

Partial differentiation with respect to T on both sides of the preceding equation yields that

$$C_\mathrm{m}\beta(\beta-1)\left(\frac{\alpha}{\mu}\right)^\beta\sum_{k=0}^{\infty}\frac{\Gamma(\beta+k)}{\Gamma(k+2)}\mathrm{e}^{-T\lambda^*}\frac{(T\lambda^*)^k}{k!}\left(T\frac{\partial\lambda^*}{\partial T}+\lambda^*\right)$$
$$= -\left(\frac{1}{\varepsilon}\right)\left(1-\frac{1}{\varepsilon}\right)\lambda_0^{1/\varepsilon}(\lambda^*)^{-1/\varepsilon-1}\frac{\partial\lambda^*}{\partial T}.$$

Assume the contrary that $\partial\lambda^*/\partial T \geq 0$. Then the left-hand side of the preceding equation was greater than zero, whereas the right-hand side was less or equal to zero. This contradiction implies that $\partial \lambda^*/\partial T < 0$ and completes the proof.

Analogously, the comparative statics properties of the optimal price p^* can be easily derived from relationship (1). Since $p = (\lambda_0/\lambda)^{1/\varepsilon}$ is strictly decreasing in λ, $\partial p^*/\partial Z$ and $\partial\lambda^*/\partial Z$ have the opposite sign, where $Z = \mu, T, C_\mathrm{m}, c, \alpha, \beta$. These arguments lead to the following analogue of Proposition 3.

Proposition 4 Suppose that $\phi(\mu) > 1 - 1/\varepsilon$. Then the optimal service price p^* determined by the first-order condition (12) is affected by the model parameters in the following manner:

(a) p^* is strictly decreasing in price elasticity ε for $p^* \geq 1$ and $\log p^* < 1/(\varepsilon - 1)$;
(b) p^* is strictly increasing in potential demand λ_0 for $\partial \log \lambda^*/\partial \log \lambda_0 < 1$;

(c) p^* is strictly decreasing in service processing rate μ;
(d) p^* is strictly increasing in the length of working period T;
(e) p^* is strictly increasing in CM cost C_m and unit operating cost c;
(f) p^* is strictly increasing in the scale parameter α and shape parameter β of facility lifetime distribution.

Proof

Properties (c)–(f) immediately follow from Proposition 3 and the price–demand relationship (1). For property (a), observation of the price–demand relationship (1) leads to $\varepsilon \log p^* = \log \lambda_0 - \log \lambda^*$ and partial differentiation with respect to ε on both sides of the preceding equation yields $(\varepsilon/p)(\partial p^*/\partial \varepsilon) = -\log p^* - (1/\lambda^*)(\partial \lambda^*/\partial \varepsilon)$. Hence, as a result of Proposition 3(a), $\partial p^*/\partial \varepsilon < 0$ if $p^* \geq 1$ and $\log p^* < 1/(\varepsilon - 1)$. This proves property (a). Finally, for property (b), partial differentiation with respect to λ_0 on both sides of $\varepsilon \log p^* = \log \lambda_0 - \log \lambda^*$ yields

$$\frac{\varepsilon}{p}\frac{\partial p^*}{\partial \lambda_0} = \frac{1}{\lambda_0} - \frac{1}{\lambda^*}\frac{\partial \lambda^*}{\partial \lambda_0} = \frac{1}{\lambda_0}\left(1 - \frac{\partial \log \lambda^*}{\partial \log \lambda_0}\right).$$

Applying Proposition 3(b), this proves property (b).

Propositions 3 and 4 have the following managerial implications for the qualitative behaviors of the optimal service demand λ^*

(a) For optimal demand λ^* sufficiently large such that $\log(\lambda_0/\lambda^*) < \varepsilon/(\varepsilon - 1)$, an increase in price elasticity ε indicates that customers are more price sensitive. Hence the service firm would reduce service price to encourage customer demand to make more profits. On the other hand, for optimal demand λ^* sufficiently small such that $\log(\lambda_0/\lambda^*) > \varepsilon/(\varepsilon - 1)$, a decrease in price elasticity ε indicates that customers are less price sensitive. Hence the service firm would raise service price to discourage customer demand to make more profits.
(b) An increase in potential demand λ_0 indicates that the market expands. Hence the service firm would accordingly reduce service price to encourage customer demand to make more profits.
(c) An increase in service processing rate μ corresponds to a decrease in facility usage per customer served, and thus it leads to lower marginal maintenance cost. To make more profits, the service firm would prefer more customers served and thus would reduce service price.
(d) Since the service facility is deteriorating, marginal maintenance cost increases over time and this trend continues as the length of working period T increases. To avoid higher marginal maintenance cost, the service firm would prefer fewer customers served and thus would raise service price.
(e) An increase in CM cost C_m (or unit operating cost c) corresponds to an increase in marginal cost. To avoid higher marginal cost, the service firm would prefer fewer customers served and thus would raise service price.
(f) Since a larger scale parameter α (or shape parameter β) corresponds to a higher (or steeper) increasing failure rate, the service facility will commit more failures and thus will incur more maintenance costs. To trade off maintenance costs for profits, the service firm would prefer fewer customers served to decrease facility failures and thus would raise service price.

Numerical examples

In this section, numerical examples are presented to illustrate the realized quantitative effects of parameter variations on the optimal pricing policy and profits. The numerically quantitative results will serve as complements to the theoretically qualitative aspects in Comparative statics analysis.

Before numerical exploits, we recall some relevant points of the profit model that are essential to numerical manipulations: (a) the service demand and facility deterioration are specified by Equations (1) and (2) respectively; and (b) the maximal expected profit is realized at the optimal demand rate given by Equation (13).

To make the numerical results more concise for discussion, several model parameters are fixed in this section: scale parameter of facility lifetime distribution $\alpha = 3.5$, mean service processing rate $\mu = 100$, potential customer arrival rate $\lambda_0 = 60$, length of working period $T = 1$, and facility setup cost $C_p = 0.2$, where the time unit is week and the monetary unit is \$1000. The resulting optimal service prices and the corresponding maximal expected profits are compared under various shape parameters ($\beta = 2, 2.5, 3$), price elasticity ($\varepsilon = 1.2, 1.4, 1.6$), minimal repair costs ($C_m = 1.0, 1.1, 1.2$), and unit operating costs ($c = 0.10, 0.15, 0.20$). The numerical manipulations are carried out by Wolfram Mathematica®. The subsequent numerical results summarized in Tables 1–3 provide the following observations.

Table 1. Effects of parameter variations for $\beta = 2$ and various combinations of ε, C_m, and c.

ε	C_m	c	p^*	λ^*	$\pi(p^*)$
1.2	1.0	0.10	1.27	44.86	49.90
		0.15	1.47	37.79	47.84
		0.20	1.69	32.06	46.10
	1.1	0.10	1.31	43.20	49.65
		0.15	1.51	36.63	47.66
		0.20	1.72	31.26	45.97
	1.2	0.10	1.35	41.72	49.42
		0.15	1.55	35.59	47.49
		0.20	1.76	30.53	45.84
1.4	1.0	0.10	0.93	66.52	49.36
		0.15	1.03	57.70	46.26
		0.20	1.14	50.07	43.57
	1.1	0.10	0.96	63.59	48.83
		0.15	1.06	55.47	45.85
		0.20	1.17	48.40	43.26
	1.2	0.10	0.99	61.01	48.33
		0.15	1.09	53.48	45.48
		0.20	1.19	46.88	42.97
1.6	1.0	0.10	0.82	83.08	50.62
		0.15	0.88	73.08	46.72
		0.20	0.96	64.16	43.30
	1.1	0.10	0.84	79.03	49.80
		0.15	0.91	69.86	46.08
		0.20	0.98	61.63	42.80
	1.2	0.10	0.87	75.49	49.05
		0.15	0.93	67.01	45.49
		0.20	1.01	59.37	42.33

Table 2. Effects of parameter variations for $\beta = 2.5$ and various combinations of ε, C_m, and c.

ε	C_m	c	p^*	λ^*	$\pi(p^*)$
1.2	1.0	0.10	1.46	38.16	49.36
		0.15	1.62	33.69	47.56
		0.20	1.80	29.68	45.98
	1.1	0.10	1.50	36.90	49.14
		0.15	1.66	32.73	47.40
		0.20	1.84	28.96	45.86
	1.2	0.10	1.54	35.78	48.94
		0.15	1.69	31.86	47.25
		0.20	1.87	28.30	45.75
1.4	1.0	0.10	1.12	51.28	47.41
		0.15	1.20	46.70	44.96
		0.20	1.28	42.36	42.73
	1.1	0.10	1.15	49.35	46.96
		0.15	1.23	45.09	44.60
		0.20	1.31	41.04	42.45
	1.2	0.10	1.18	47.65	46.56
		0.15	1.26	43.66	44.28
		0.20	1.34	39.85	42.19
1.6	1.0	0.10	1.00	59.88	47.01
		0.15	1.05	55.24	44.13
		0.20	1.11	50.75	41.48
	1.1	0.10	1.03	57.46	46.36
		0.15	1.08	53.15	43.60
		0.20	1.14	48.96	41.05
	1.2	0.10	1.05	55.33	45.78
		0.15	1.10	51.29	43.12
		0.20	1.16	47.36	40.65

(1) Service price p^* increases (customer demand λ^* decreases) as the shape parameter β of the facility lifetime distribution increases. This indicates that the service firm should charge a higher service price provided the facility has a steeper increasing failure rate. Hence service price should be raised to discourage customer demand.
(2) Service price p^* decreases (customer demand λ^* increases) as the price elasticity ε increases. This indicates that the service firm should charge a lower service price provided the customers are more price sensitive. Hence service price should be reduced to encourage customer demand.
(3) Service price p^* increases (customer demand λ^* decreases) as the minimal repair cost C_m increases. This indicates that the service facility with a higher minimal repair cost should serve fewer customers to reduce facility failures. Hence service price is raised to discourage customer demand.
(4) Service price p^* increases (customer demand λ^* decreases) as the unit operating cost c increases. This indicates a consequence of the trade-off between operating costs and service revenues. Hence service price is raised to discourage customer demand.

The procedure of sensitivity analysis can be similarly conducted on other model parameters α, λ_0, μ, and T, which also affect the optimal pricing policy as described in Proposition 3. The quantitative results of the presented sensitivity analysis are informative

Table 3. Effects of parameter variations for $\beta = 3$ and various combinations of ε, C_m, and c.

ε	C_m	c	p^*	λ^*	$\pi(p^*)$
1.2	1.0	0.10	1.60	34.07	48.99
		0.15	1.74	30.95	47.36
		0.20	1.89	27.94	45.89
	1.1	0.10	1.64	33.07	48.80
		0.15	1.78	30.14	47.22
		0.20	1.93	27.30	45.78
	1.2	0.10	1.68	32.19	48.62
		0.15	1.81	29.41	47.08
		0.20	1.96	26.72	45.68
1.4	1.0	0.10	1.27	43.13	46.16
		0.15	1.33	40.29	44.07
		0.20	1.40	37.46	42.13
	1.1	0.10	1.30	41.73	45.78
		0.15	1.36	39.06	43.76
		0.20	1.43	36.40	41.88
	1.2	0.10	1.32	40.49	45.44
		0.15	1.39	37.96	43.48
		0.20	1.46	35.45	41.64
1.6	1.0	0.10	1.14	48.53	44.84
		0.15	1.18	45.82	42.48
		0.20	1.23	43.09	40.26
	1.1	0.10	1.17	46.87	44.31
		0.15	1.21	44.31	42.04
		0.20	1.25	41.74	39.88
	1.2	0.10	1.19	45.40	43.84
		0.15	1.23	42.98	41.63
		0.20	1.28	40.55	39.54

for the service firm to adjust its original optimal pricing policy according to parameter variations.

Conclusion

This paper deals with the pricing policies of services with considerable facility maintenance costs under log-linear service demand and Weibull facility lifetime. Under the model assumptions, a mathematical profit model of the service firm is developed and investigated.

Theoretical implications

The most important theoretical finding in this paper is the existence of a unique optimal pricing policy. As the facility lifetime has an increasing failure rate and the service demand is price elastic, it is intuitive that the marginal maintenance cost is increasing in customer arrival rate. Therefore, the conditions of optimality indicate a trade-off between service revenues and maintenance costs.

It is also found from the mathematical expression that the expected number of facility failures in this intermittently operated service system is greater than the one in the continuously operated environment. This finding is a consequence of system risk. Because customer arrivals and service times are stochastic, the service facility faces higher uncertainty under the intermittent operation. This explains the origin of the additional system risk.

Finally, the condition that characterizes the optimal pricing policy is found to be moderately sophisticated. This complexity is due to the presence of facility maintenance costs and the intrinsic properties of an intermittently operated service system. This suggests us to find the optimal policy by numerical methods rather than by closed-form expression.

Managerial implications

Pricing is a powerful weapon to attract customers. However, more customers do not necessarily bring more profits to the service firm. The findings of this study suggest that, as the service system is subject to facility maintenance costs, the system manager should examine the bidirectional relationship between customer demand and facility deterioration to determine the service price.

This study also suggests that the system manager should be aware of the changes of service demand, service restrictions, cost structure, scale/shape of facility lifetime, etc. The sensitivity analysis shows the impacts of parameter variations on the optimal pricing policy and indicates that the service price should be adjusted to reflect the changes of business environment.

Limitations and future research

We will discuss several limitations in the present study and, in the meanwhile, provide suggestions to extend the present framework. First, customer arrivals are assumed to be governed by a homogeneous Poisson process. However, they might fluctuate from time to time. Therefore, a nonhomogeneous Poisson demand would be more appropriate under this nonstationary condition. By employing a time-dependent demand, our model can be extended and the original static pricing problem turns into a dynamic pricing problem.

Second, CM cost is assumed to be fixed. However, CM cost might increase as facility deterioration gets worse over time. Therefore, a variable CM cost would be more flexible. The most practical approach is to use a linear CM cost of the form $a + bx$, where $a,b > 0$. In this cost function, a and b, respectively, denote the fixed cost and variable cost of a CM action, and x might represent the cumulative operating time of the service facility.

Third, the presence of strategic customers is not considered. The customers might have avoid-the-crowd or follow-the-crowd behavior. Customers who want to reduce the time spent in the system would turn away to avoid the crowd when the waiting line is long. This phenomenon gives a suggestion to adopt the notion of actual arrival rate to amend the original arrival rate. In terms of technicalities, the arrival rate can be obtained by considering the probability of customer balking that is dependent on the length of the waiting line. For a high-quality service whose value is indicated by the length of the waiting line, customers would accept the information signaled by the crowd and longer waiting lines would be formed. However, a long waiting line implies more waiting time. This herd behavior shows a need to consider a framework in which customers choose priority levels to be purchased.

Finally, the effect of waiting room capacity is ignored. When waiting room capacity is small, many customers are lost even if they are willing to pay the price. On the other hand, when the capacity is large, the incurred capacity costs are not deserved. By adding a decision variable denoting waiting room capacity to the original service demand, customer sensitivity to waiting room capacity could be well reflected. Besides, capacity costs could be incorporated into the profit model to help evaluate the investment on waiting room capacity. This discussion inspires an investigation on joint policies of service price and waiting room capacity.

References

Barlow, R.E., & Hunter, L. (1960). Optimum preventive maintenance policies. *Operations Research*, *8,* 90–100.

Barlow, R.E., & Proschan, F. (1965). *Mathematical theory of reliability*. New York, NY: Wiley.

Brynjolfsson, E., Hu, Y.J., & Smith, M.D. (2003). Consumer surplus in the digital economy: Estimating the value of increased product variety at online booksellers. *Management Science*, *49,* 1580–1596.

Dohi, T., Kaio, N., & Osaki, S. (2001). Optimal periodic maintenance strategy under an intermittently used environment. *IIE Transactions*, *33,* 1037–1046.

Gyldmark, M., & Morrison, G.C. (2001). Demand for health care in Denmark: Results of a national sample survey using contingent valuation. *Social Science & Medicine*, *53,* 1023–1036.

Hassin, R., & Haviv, M. (2003). *To queue or not to queue: Equilibrium behavior in queueing systems*. Boston, MA: Kluwer.

Hsu, F. (1992). Optimal preventive maintenance policies in an *M*/*G*/1 queue-like production system. *European Journal of Operational Research*, *58,* 112–122.

Hsu, F. (1999). Simultaneous determination of preventive maintenance and replacement policies in a queue-like production system with minimal repair. *Reliability Engineering & System Safety*, *63,* 161–167.

Johnson, N.L., Kemp, A.W., & Kotz, S. (2005). *Univariate discrete distributions* (3rd ed., Chapter 4, pp. 156–207). New York, NY: Wiley.

Koyanagi, J., & Kawai, H. (2003). An optimal age maintenance for an *M*/*G*/1 queueing system. *Mathematical and Computer Modelling*, *38,* 1333–1338.

Lanning, S.G., Mitra, D., Wang, Q., & Wright, M.H. (2000). Optimal planning for optical transport networks. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, *358,* 2183–2196.

Lawless, J.F. (2003). *Statistical models and methods for lifetime data* (2nd ed., Chapter 1, pp. 1–47). New York, NY: Wiley.

Nakagawa, T., & Kowada, M. (1983). Analysis of a system with minimal repair and its application to replacement policy. *European Journal of Operational Research*, *12,* 176–182.

Scarf, P.A. (1997). On the application of mathematical models in maintenance. *European Journal of Operational Research*, *99,* 493–506.

So, K.C., & Song, S. (1998). Price, delivery time guarantees and capacity selection. *European Journal of Operational Research*, *111,* 28–49.

Wang, H. (2002). A survey of maintenance policies of deteriorating systems. *European Journal of Operational Research*, *139,* 469–489.

Ziya, S., Ayhan, H., & Foley, R.D. (2006). Optimal prices for finite capacity queueing systems. *Operations Research Letters*, *34,* 214–218.